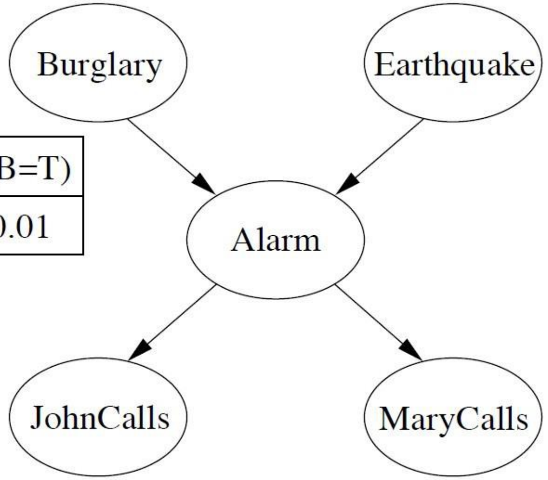
Example

- Query Analysis:
  - Query *variables*: Earthquake
  - *Evidence* (*observed*) variables and their values: JohnCalls, MaryCalls
  - *Unobserved* (hidden/latent) variables: Burglary, Alarm
- $P(E|j,m) = \alpha P(E,j,m)$
- $P(E,j,m) = \sum_a \sum_b P(E,j,m,b,a)$
  - marginalisation of all possible values of *A* and *B*
- $P(E,j,m) =$
  $\sum_a \sum_b P(b)\ P(E)\ P(a|b,E)\ P(j|a)\ P(m|a)$



P(B=T)
0.01

P(E=T)
0.02

B	E	P(A=T|B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J=T|A)
T	0.90
F	0.05

A	P(M=T|A)
T	0.70
F	0.01

Compute the probability that there is an earthquake given both John and Mary call.
$P\ (E = T\ |\ J = T, M = T) = ?$

Find the solution through inference.
Approaches to Inference (state estimation):

- Enumeration
- Variable elimination

2- Enumeration Approach: better solution

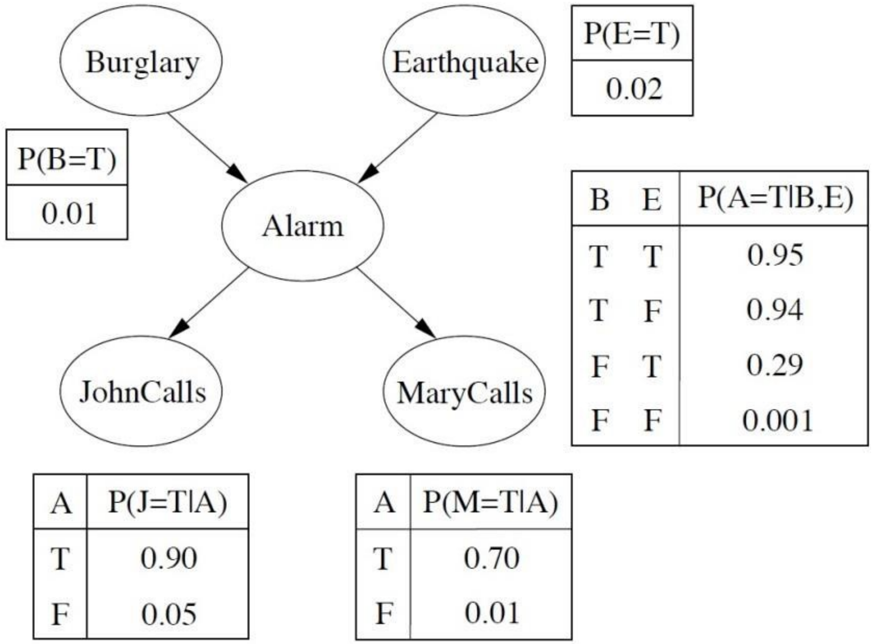
- A form of dynamic programming approach
  - Using *factor tables* to store the immediate results
- Two key operations:
  - Multiplication
  - Marginalisation

The initial factor tables are the reformatted CPTs:

B	$f_B(B)$
T	0.01
F	0.99

E	$f_E(E)$
T	0.02
F	0.98

A	B	E	$f_A(A,B,E)$
T	T	T	0 .95
T	T	F	0 .94
T	F	T	0 .29
T	F	F	0 .001
F	T	T	0 .05
F	T	F	0 .06
F	F	T	0 .71
F	F	F	0 .999



A	$f_J(A)$
T	0.9
F	0.05

A	$f_M(A)$
T	0.7
F	0.01

Query Analysis:

$$P(E) \sum_{b} P(b) \sum_{a} P(a|b,E)\, P(j|a)\, P(m|a)$$

$$f_E(E) \sum_{b} f_B(B) \sum_{a} f_A(A,B,E) f_J(A) f_M(A)$$

Variable Elimination Algorithm:

- Bottom-up computations
- Step1: $f_E(E) \sum_b f_B(B) \sum_a f_A(A,B,E) f_J(A) f_M(A)$
- $f_{JM}(A) = f_J(A) f_M(A)$ → Multiplication

A	$f_{JM}(A)$
T	$.9 \times .7$
F	$.05 \times .01$

=

A	$f_J(A)$
T	0.9
F	0.05

×

A	$f_M(A)$
T	0.7
F	0.01

=

A	$f_{JM}(A)$
T	.63
F	.0005

B	$f_B(B)$
T	0.01
F	0.99

E	$f_E(E)$
T	0.02
F	0.98

A	B	E	$f_A(A,B,E)$
T	T	T	0 .95
T	T	F	0 .94
T	F	T	0 .29
T	F	F	0 .001
F	T	T	0 .05
F	T	F	0 .06
F	F	T	0 .71
F	F	F	0 .999

A	$f_J(A)$
T	0.9
F	0.05

A	$f_M(A)$
T	0.7
F	0.01

Variable Elimination Algorithm:

- Step2: $f_E(\mathrm{E}) \sum_{\mathrm{b}} f_B(\mathrm{B}) \sum_{\mathrm{a}} f_A(\mathrm{A,B,E}) f_{JM}(\mathrm{A})$
- $f_{AJM}(\mathrm{A,B,E}) = f_A(\mathrm{A,B,E}) f_{JM}(\mathrm{A})$ → Multiplication

B	$f_B(B)$
T	0.01
F	0.99

E	$f_E(E)$
T	0.02
F	0.98

A	B	E	$f_A(A,B,E)$
T	T	T	0 .95
T	T	F	0 .94
T	F	T	0 .29
T	F	F	0 .001
F	T	T	0 .05
F	T	F	0 .06
F	F	T	0 .71
F	F	F	0 .999

A	$f_{JM}(A)$
T	.63
F	.0005

Step2

A	B	E	$f_{AJM}(A,B,E)$
T	T	T	$.95 \times .63$
T	T	F	$.94 \times .63$
T	F	T	$.29 \times .63$
T	F	F	$.001 \times .63$
F	T	T	$.05 \times .0005$
F	T	F	$.06 \times .0005$
F	F	T	$.71 \times .0005$
F	F	F	$.999 \times .0005$

Step3

B	E	$f_{\underline{A}JM}(A,B,E)$
T	T	$.95 \times .63 + .05 \times .0005$
T	F	$.94 \times .63 + .06 \times .0005$
F	T	$.29 \times .63 + .71 \times .0005$
F	F	$.001 \times .63 + .999 \times .0005$

- Step3: $f_E(\mathrm{E}) \sum_{\mathrm{b}} f_B(\mathrm{B}) \sum_{\mathrm{a}} f_{AJM}(\mathrm{A,B,E})$
- $f_{\underline{A}JM}(\mathrm{B,E}) = \sum_{\mathrm{a}} f_{AJM}(\mathrm{A,B,E})$ → Marginalisation

A	$f_J(A)$
T	0.9
F	0.05

A	$f_M(A)$
T	0.7
F	0.01

Variable Elimination Algorithm:

- Step4: f_E(E) $\sum_b f_B$(B)$f_{\underline{A}JM}$(B,E)
- $f_{B\underline{A}JM}$(B,E) = f_B(B)$f_{\underline{A}JM}$(B,E) ⟶ Multiplication

Step4

B	$f_B(B)$
T	0.01
F	0.99

×

B	E	$f_{AJM}(A,B,E)$
T	T	.5985
T	F	.5922
F	T	.183
F	F	.001129

=

B	E	$f_{B\underline{A}JM}(B,E)$
T	T	.01 × .5985
T	F	.01 × .5922
F	T	.99 × .183
F	F	.99 × .001129

- In the same way
- Step5: Marginalisation over b $f_{B\underline{A}JM}$(E)
- Step6: Multiplication $f_{E\underline{BA}JM}$(E)

Step5

E	$f_{B\underline{A}JM}(B,E)$
T	.01 × .5985 + .99 × .183 = 0.1872
F	.01 × .5922 + .99 × .001129 = 0.0070

×

E	$f_E(E)$
T	0.02
F	0.98

=

E	$f_{EB\underline{A}JM}(E)$
T	.02 × .1872= .0037
F	.98 × .0070= .0069

- Last Step: normalisation: $P(E|j,m) = \frac{0.0037}{0.0037+0.0069} = 0.3491$