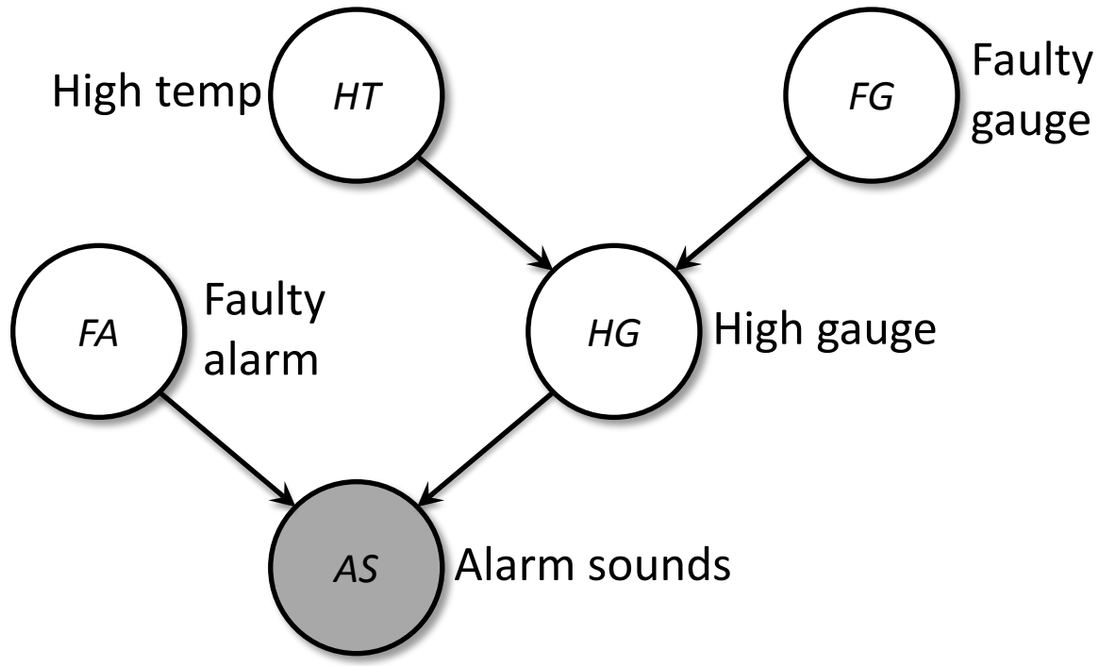
COMP90051

Elimination Algorithm

Nuclear power plant

- **Alarm sounds**; meltdown?!



- $\Pr(HT|AS=t) = \dfrac{\Pr(HT, AS=t)}{\Pr(AS=t)}$

$$= \frac{\sum_{FG,HG,FA} \Pr(AS=t, FA, HG, FG, HT)}{\sum_{FG,HG,FA,HT'} \Pr(AS=t, FA, HR, FG, HT')}$$

- Numerator (denominator similar)

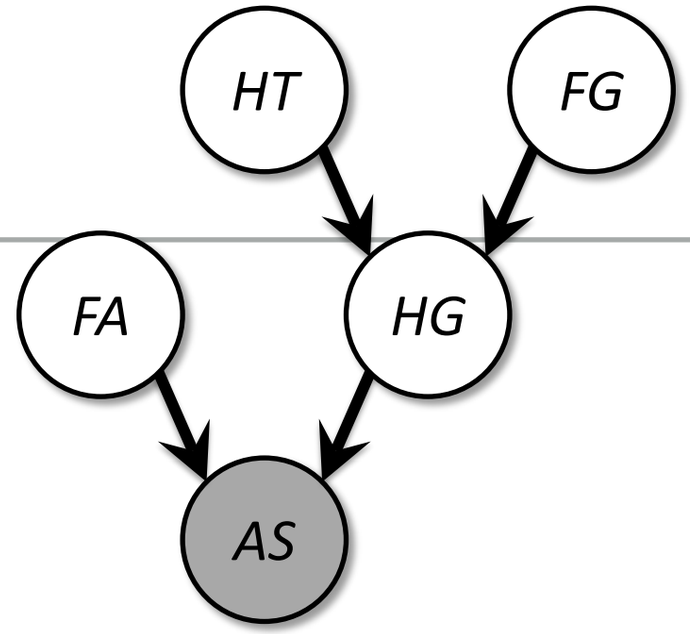
expanding out sums, joint *summing once over 2^5 table*

$$= \sum_{FG} \sum_{HG} \sum_{FA} \Pr(HT) \Pr(HG|HT, FG) \Pr(FG) \Pr(AS=t|FA, HG) \Pr(FA)$$

distributing the sums as far down as possible *summing over several smaller tables*

$$= \Pr(HT) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA) \Pr(AS=t|FA, HG)$$

To calculate $P(HT|AS = 1)$



- Joint

$$P(AS, FA, HG, HT, FG)$$
$$= P(AS|FA, HG)P(FA)P(HG|HT, FG)P(HT)P(FG)$$

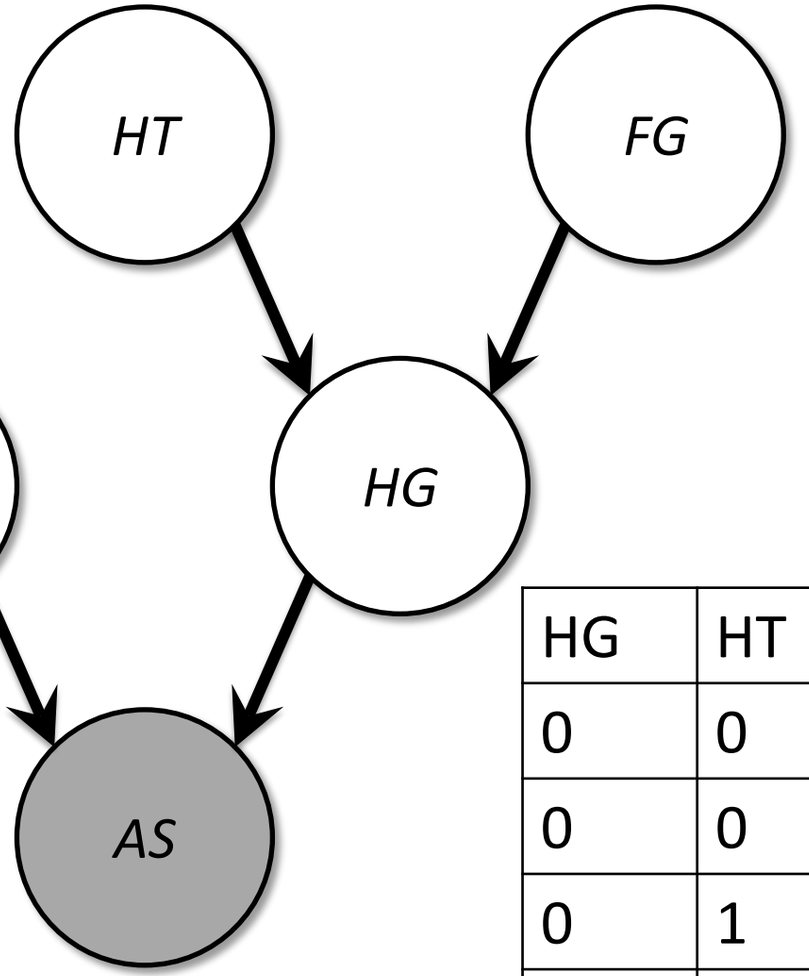
- Step 1. $P(HT|AS = 1) \propto P(AS = 1, HT)$
- Step 2. $P(AS = 1, HT) = \sum_{FG,HG,FA} P(AS = 1, FA, HG, HT, FG)$
- Step 3. Normalize $P(AS = 1, HT) \rightarrow P(HT|AS = 1)$

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2

FG	P(FG)
0	0.9
1	0.1

FA	P(FA)
0	0.7
1	0.3

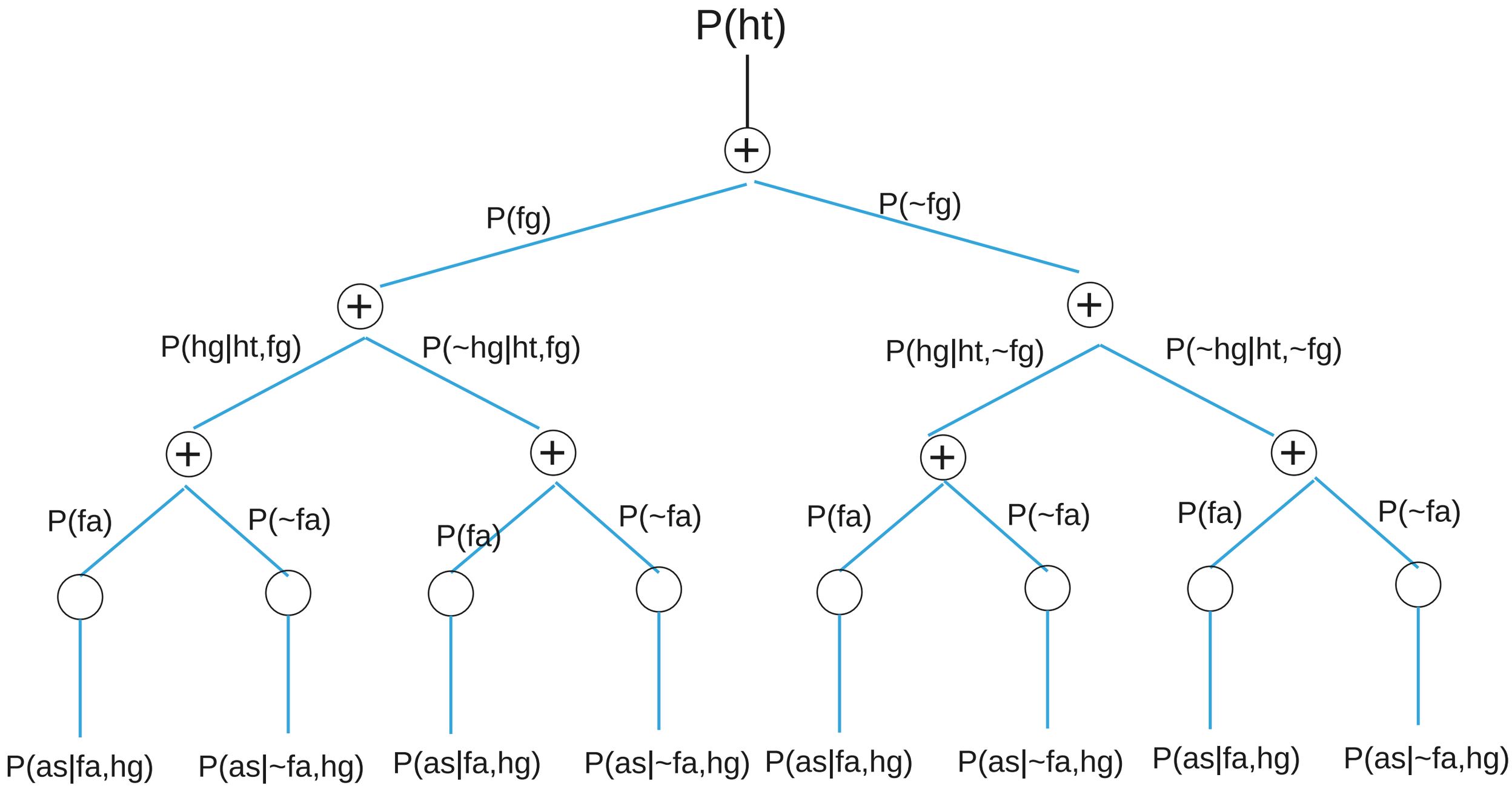


AS	FA	HG	P(AS|FA,HG)
0	0	0	0.9
0	0	1	0.1
0	1	0	0.6
0	1	1	0.4
1	0	0	0.1
1	0	1	0.9
1	1	0	0.4
1	1	1	0.6

HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

P(HT=1|AS=1): Numerator =

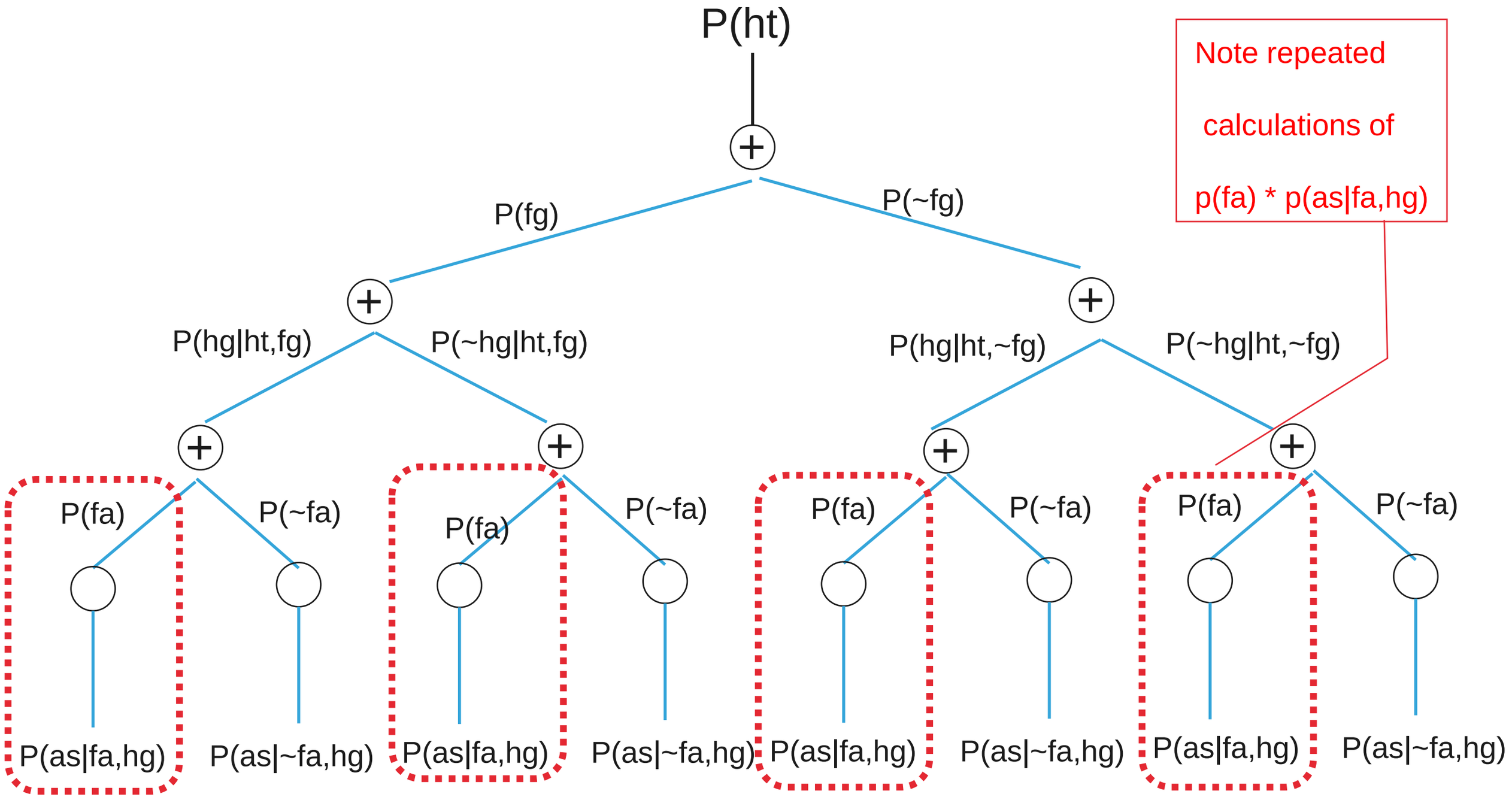
$$\Pr(HT = 1) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA) \Pr(AS = t|FA, HG)$$



KEY: FA is a random variable which can be either 0 or 1; P(fa) is P(FA = 1); p(~fa) is P(FA = 0)

P(HT=1|AS=1): Numerator =

$$\Pr(HT = 1) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA) \Pr(AS = t|FA, HG)$$



KEY: FA is a random variable which can be either 0 or 1; P(fa) is P(FA = 1); p(~fa) is P(FA = 0)

Elimination Algorithm

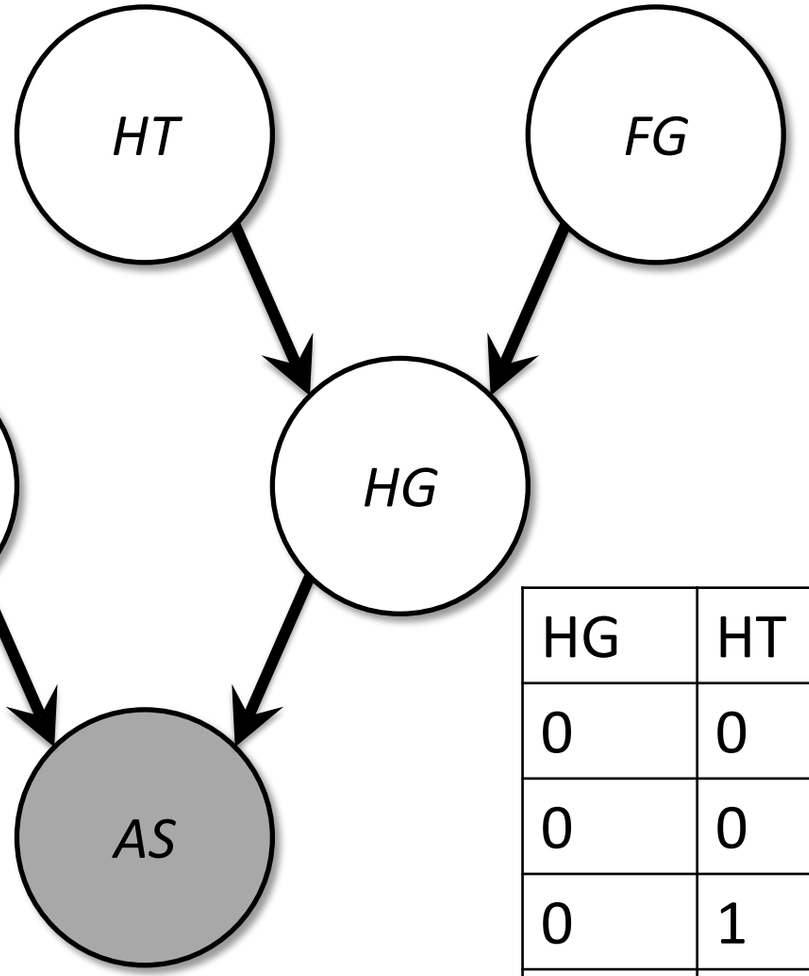
- This is the case with multiple other calculations
- Instead of computing them again and again, we compute individual calculations in tables, and lookup values.

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2

FG	P(FG)
0	0.9
1	0.1

FA	P(FA)
0	0.7
1	0.3



AS	FA	HG	P(AS|FA,HG)
0	0	0	0.9
0	0	1	0.1
0	1	0	0.6
0	1	1	0.4
1	0	0	0.1
1	0	1	0.9
1	1	0	0.4
1	1	1	0.6

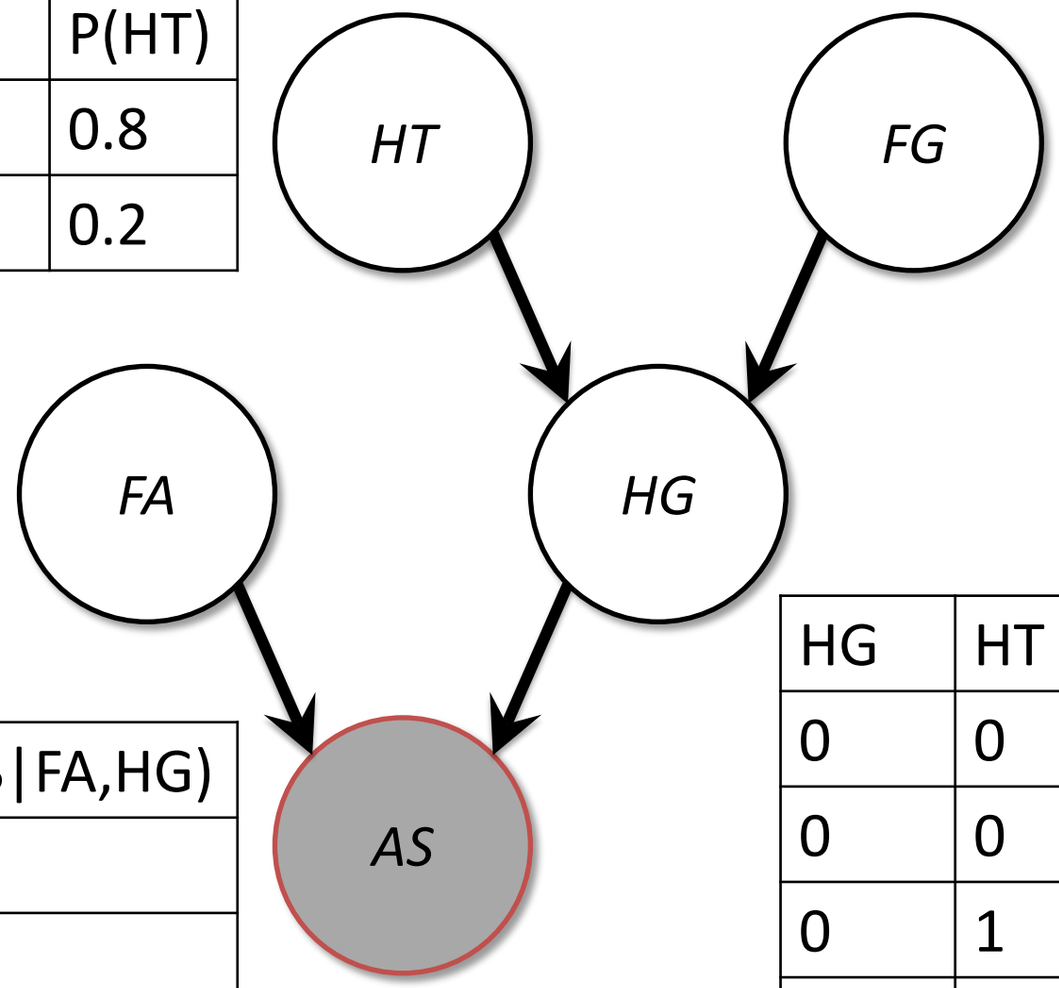
HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2

FG	P(FG)
0	0.9
1	0.1

FA	P(FA)
0	0.7
1	0.3



STEP 1: We know that AS = 1

AS	FA	HG	P(AS|FA,HG)
~~0~~	~~0~~	~~0~~	~~0.9~~
~~0~~	~~0~~	~~1~~	~~0.1~~
~~0~~	~~1~~	~~0~~	~~0.6~~
~~0~~	~~1~~	~~1~~	~~0.4~~
1	0	0	0.1
1	0	1	0.9
1	1	0	0.4
1	1	1	0.6

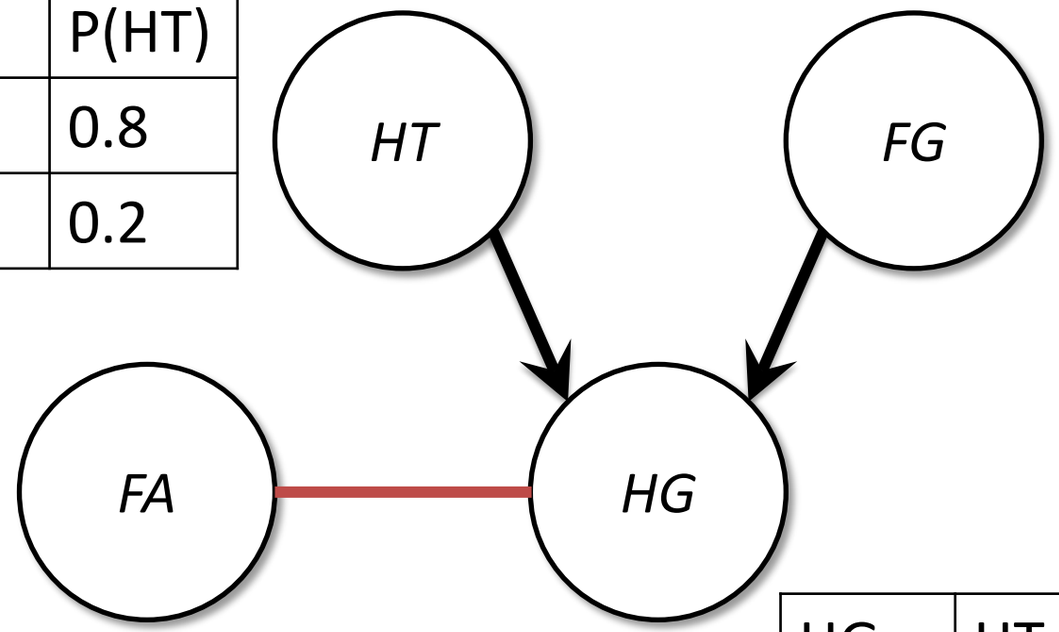
HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2

FG	P(FG)
0	0.9
1	0.1

FA	P(FA)
0	0.7
1	0.3



HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

STEP 1: We know that AS = 1. So m_AS(FA,HG) = P(AS|FA,HG) with AS = 1

M_AS(FA,HG)

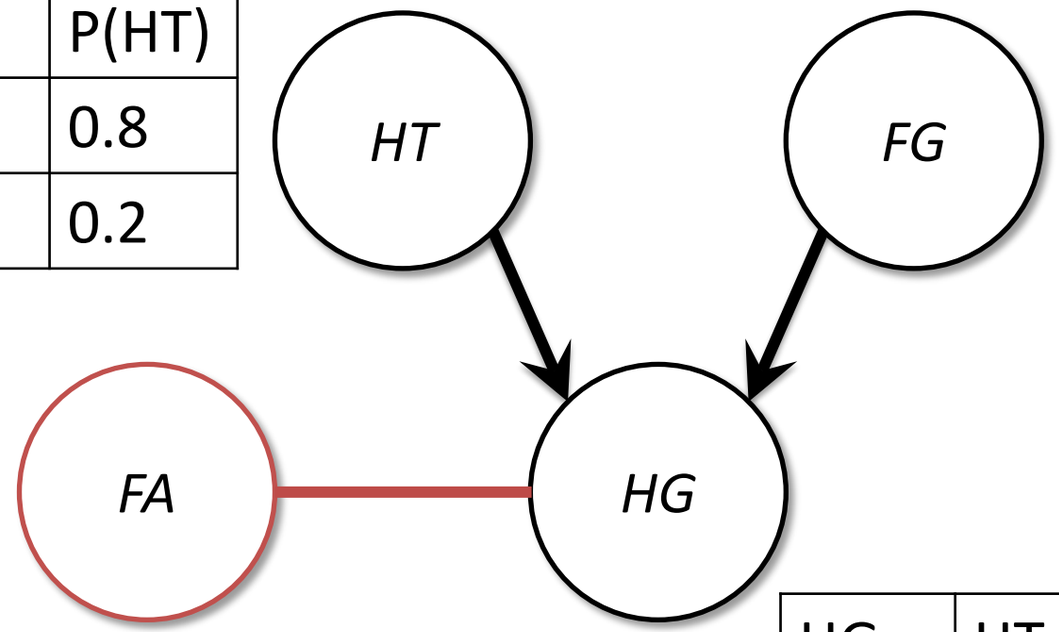
FA	HG	M_AS(FA,HG)
0	0	0.1
0	1	0.9
1	0	0.4
1	1	0.6

Nuclear Power Plant

STEP 2: Eliminating FA

HT	P(HT)
0	0.8
1	0.2

FG	P(FG)
0	0.9
1	0.1



FA	P(FA)
0	0.7
1	0.3

STEP 2: Eliminating FA, and defining

M_FA(HG) = $\sum_{FA}$ p(FA) * m_AS(FA,HG)

FA	HG	M_AS(FA,HG)
0	0	0.1
0	1	0.9
1	0	0.4
1	1	0.6

HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

Nuclear Power Plant

p(FA) * m_AS(FA,HG)

FA	P(FA)
0	0.7
1	0.3

*

FA	HG	M_AS(FA,HG)
0	0	0.1
0	1	0.9
1	0	0.4
1	1	0.6

=

FA	HG	p(FA) * M_AS(FA,HG)
0	0	0.1 * 0.7 = 0.07
0	1	0.9 * 0.7 = 0.63
1	0	0.4 * 0.3 = 0.12
1	1	0.6 * 0.3 = 0.24

M_FA(HG) = $\sum_{FA}$ p(FA) * m_AS(FA,HG)

FA	HG	p(FA) * M_AS(FA,HG)
0	0	0.1 * 0.7 = 0.07
0	1	0.9 * 0.7 = 0.63
1	0	0.4 * 0.3 = 0.12
1	1	0.6 * 0.3 = 0.24

HG	M_FA(HG)
0	0.07 + 0.12 = 0.19
1	0.63 + 0.24 = 0.81

Nuclear Power Plant

Calculating the same thing in one step

$$M\_FA(HG) = \sum_{FA} p(FA) * m\_AS(FA,HG)$$

FA	P(FA)
0	0.7
1	0.3

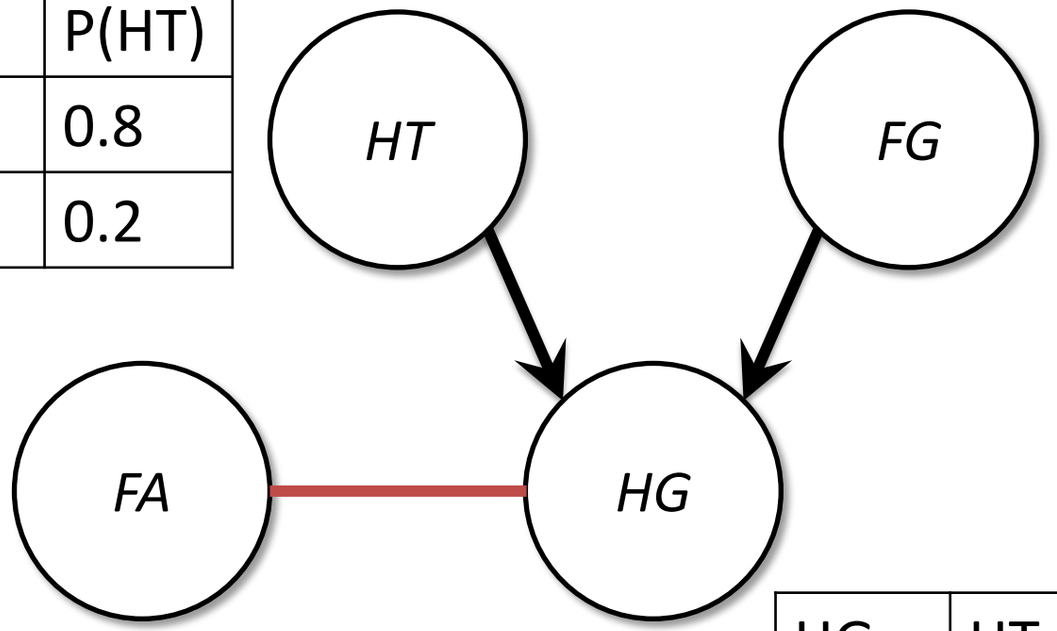
*

FA	HG	M_AS(FA,HG)
0	0	0.1
0	1	0.9
1	0	0.4
1	1	0.6

HG	M_FA(HG)
0	0.1 * 0.7 + 0.4 * 0.3 = 0.19
1	0.9 * 0.7 + 0.6 * 0.3 = 0.81

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2



FG	P(FG)
0	0.9
1	0.1

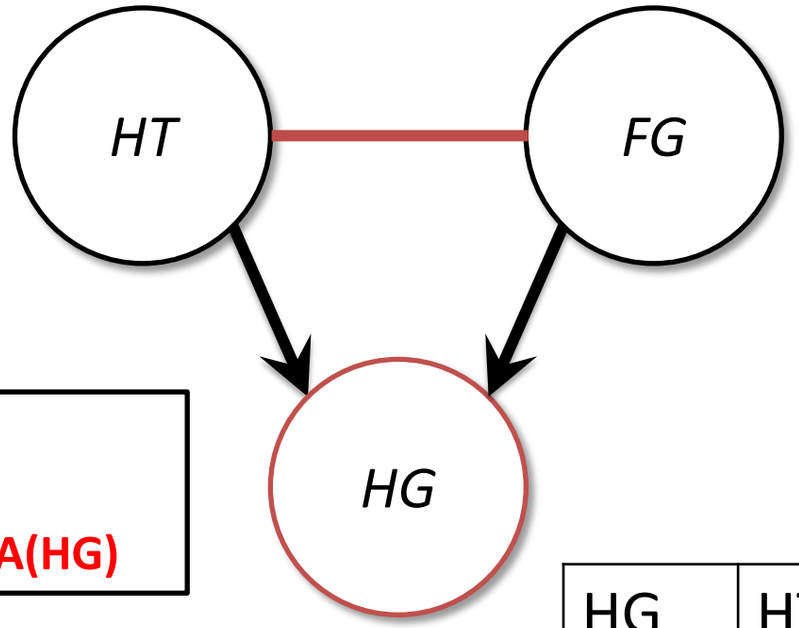
M_FA(HG)

HG	M_FA(HG)
0	0.19
1	0.81

HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2



FG	P(FG)
0	0.9
1	0.1

STEP 3: Eliminating HG: Define

M_HG(HT,FG) = $\sum_{HG}$ P(HG|HT,FG) * M_FA(HG)

HG	M_FA(HG)
0	0.19
1	0.81

HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

P(HG|HT,FG) * M_FA(HG)

HG	HT	FG	P(HG|HT,FG)
0	0	0	0.8
0	0	1	0.6
0	1	0	0.1
0	1	1	0.3
1	0	0	0.2
1	0	1	0.4
1	1	0	0.9
1	1	1	0.7

*

HG	M_FA(HG)
0	0.19
1	0.81

=

HG	HT	FG	P(HG|HT,FG) * M_FA(HG)
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

M_HG(HT,FG) = $\sum_{HG}$ P(HG|HT,FG) * M_FA(HG)

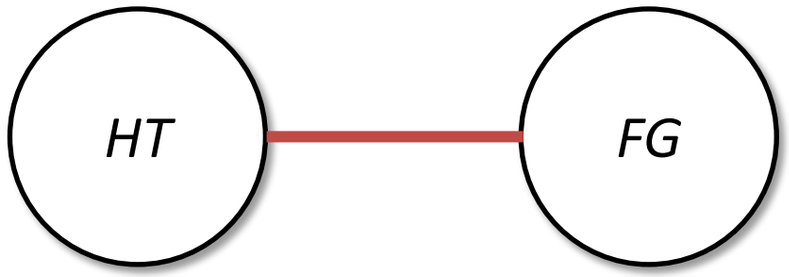
HG	HT	FG	P(HG|HT,FG) * M_FA(HG)
0	0	0	.152
0	0	1	.114
0	1	0	.019
0	1	1	.057
1	0	0	.162
1	0	1	.324
1	1	0	.729
1	1	1	.567

= M_HG(HT,FG)

HT	FG	M_HG(HT,FG)
0	0	?
0	1	?
1	0	?
1	1	?

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2



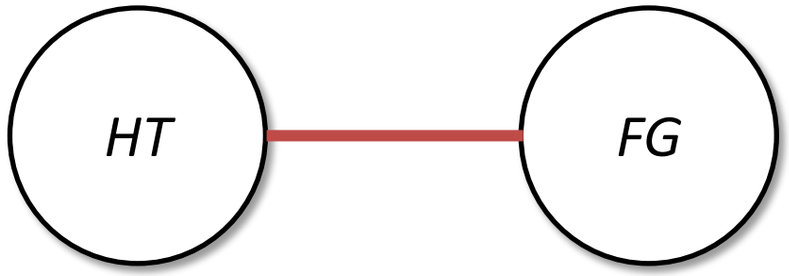
FG	P(FG)
0	0.9
1	0.1

M_HG(HT,FG)

HT	FG	M_HG(HT,FG)
0	0	0.314
0	1	0.438
1	0	0.748
1	1	0.624

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2



FG	P(FG)
0	0.9
1	0.1

M_HG(HT,FG)

HT	FG	M_HG(HT,FG)
0	0	0.314
0	1	0.438
1	0	0.748
1	1	0.624

STEP 4: Eliminating FG: Define

M_FG(HT) = $\sum_{HT}$ P(FG) * M_HG(HT,FG)

Nuclear Power Plant

P(FG) * M_HG(HT,FG)

FG	P(FG)
0	0.9
1	0.1

*

HT	FG	M_HG(HT,FG)
0	0	0.314
0	1	0.438
1	0	0.748
1	1	0.624

HT	FG	P(FG) * M_FG(HT)
0	0	?
0	1	?
1	0	?
1	1	?

M_FG(HT) = $\sum_{HT}$ P(FG) * M_HG(HT,FG)

HT	M_FG(HT)
0	?
1	?

Nuclear Power Plant

M_FG(HT) = P(FG) * M_HG(HT,FG)

FG	P(FG)
0	0.9
1	0.1

*

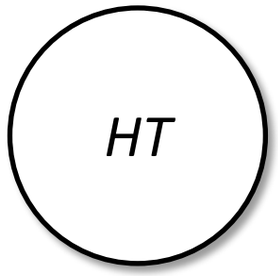
HT	FG	M_HG(HT,FG)
0	0	0.314
0	1	0.438
1	0	0.748
1	1	0.624

= M_FG(HT)

HT	M_FG(HT)
0	0.9 * 0.314 + 0.1 * 0.438
1	?

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2

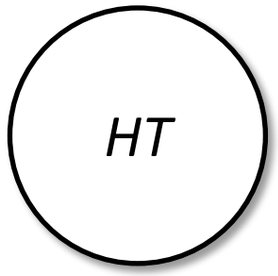


M_FG(HT)

HT	M_FG(HT)
0	0.3264
1	0.7356

Nuclear Power Plant

HT	P(HT)
0	0.8
1	0.2



M_FG(HT)

HT	M_FG(HT)
0	0.3264
1	0.7356

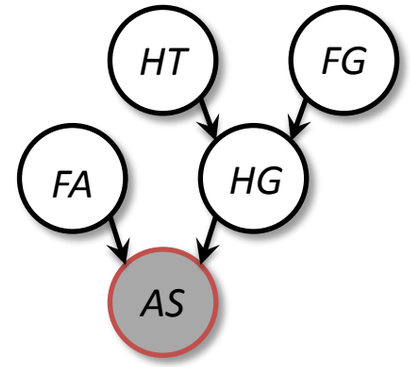
STEP 5: Calculating Numerator of P(HT| AS = 1)

Numerator of P(HT = 0|AS=1) = P(HT=0) * M_FG(HT =0)

= 0.8 * 0.3264 = 0.14712

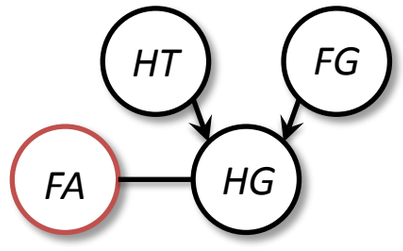
Numerator of P(HT = 1|AS=1) = ?

Nuclear power plant:Summary



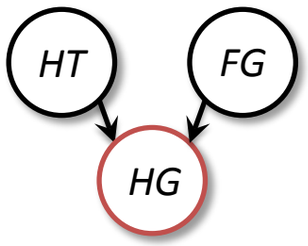
$$= \Pr(HT) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA) \Pr(AS = t|FA, HG)$$

eliminate *AS*: since *AS* observed, really a no-op



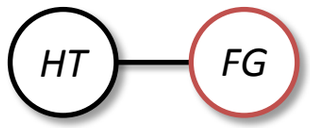
$$= \Pr(HT) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG) \sum_{FA} \Pr(FA)\, m_{AS}(FA, HG)$$

eliminate *FA*: multiplying 1x2 by 2x2



$$= \Pr(HT) \sum_{FG} \Pr(FG) \sum_{HG} \Pr(HG|HT, FG)\, m_{FA}(HG)$$

eliminate *HG*: multiplying 2x2x2 by 2x1



$$= \Pr(HT) \sum_{FG} \Pr(FG)\, m_{HG}(HT, FG)$$

eliminate *FG*: multiplying 1x2 by 2x2

$$= \Pr(HT)\, m_{FG}(HT)$$