Probabilistic Graphical Models

SML 17

Workshop#10

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Agenda

- Probabilistic Graphical Models
  - Worksheet solution
    - Design PGMs
    - PGMs: Joint likelihood
      - Using chain rule
      - Using conditional independence assumptions
    - PGMs: # of parameters
      - Using Conditional PTs (Probability Tables)
      - Using Full joint PTs
    - PGMs: Query Answer
    - PGMs: Conditional Independence
  - PGMs: Inference Discussion
    - Queries: prediction, diagnosis, learning
    - Inference Approaches:
      - Enumeration
      - Variable Elimination

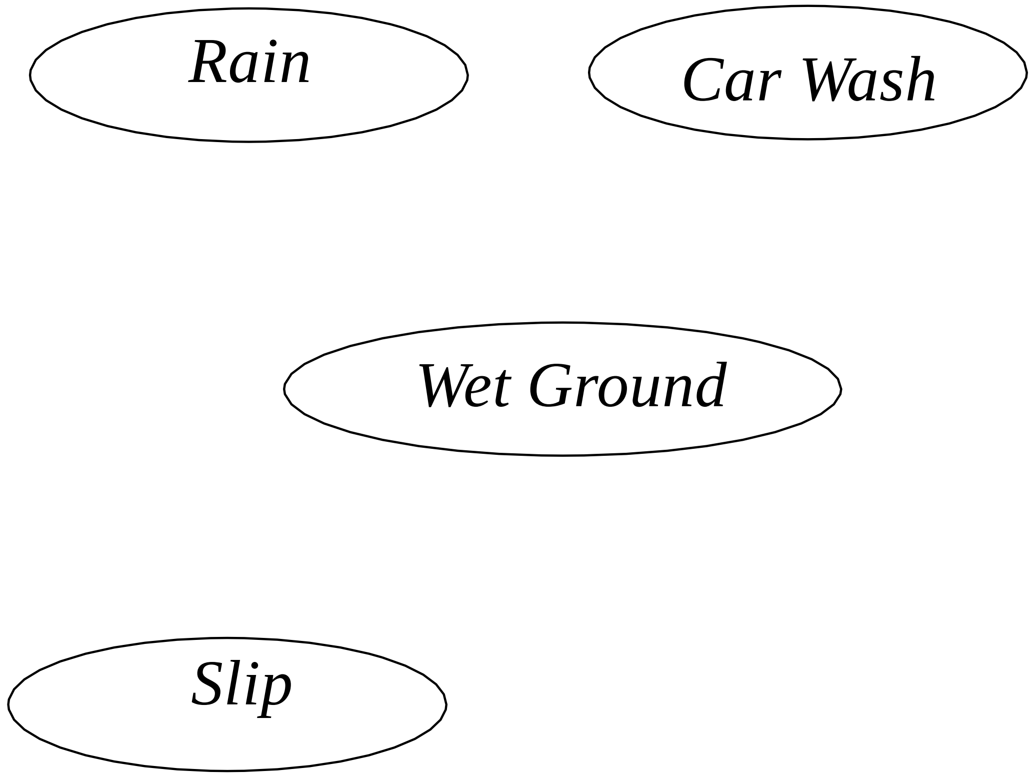
Worksheet: Q1- Design PGM

- Draw a PGD to model the following scenario. Consider the problem of a robot slipping while walking in a street searching for a specific object. The slip is based on the ground being wet and there is two factors cause the ground to be wet: having rain and/or washing cars in the street. You will need to think about what RVs are needed.

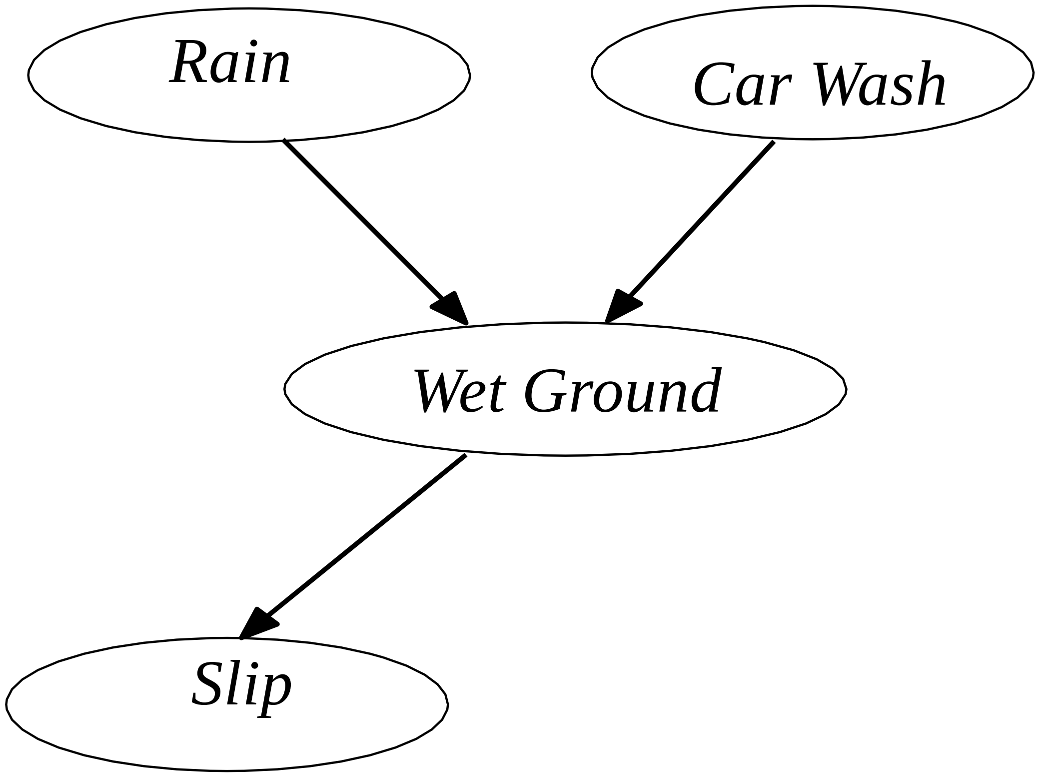
Tips:

- Think of how many nodes should be included?
- Think of the dependencies for each node (i.e. r.v. or event) ?

Worksheet: Q1- Design PGM - solution

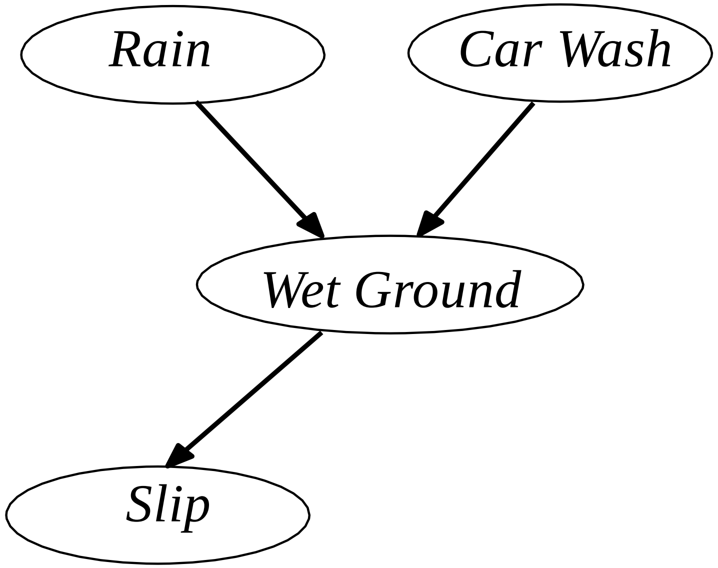


Worksheet: Q1- Design PGM - solution



Worksheet: Q2- joint likelihood PGM

- Write the factorised joint distribution according to the designed graph.

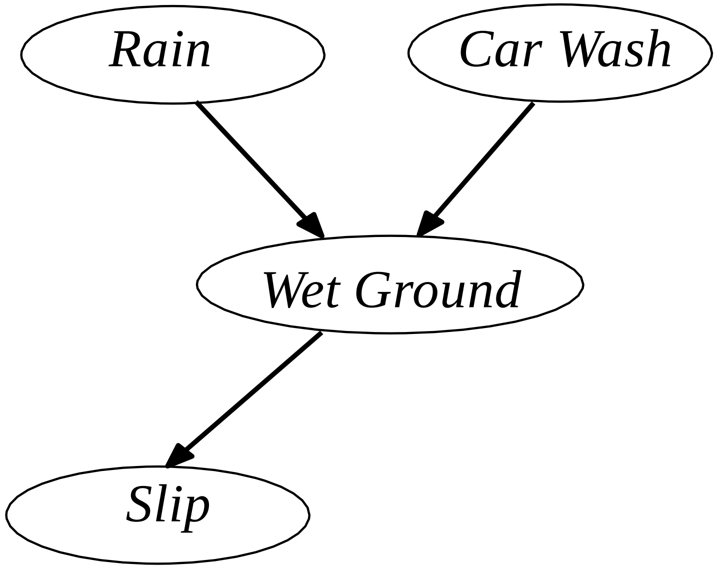


Worksheet: Q2- joint likelihood - solution

- Write the factorised joint distribution according to the designed graph.

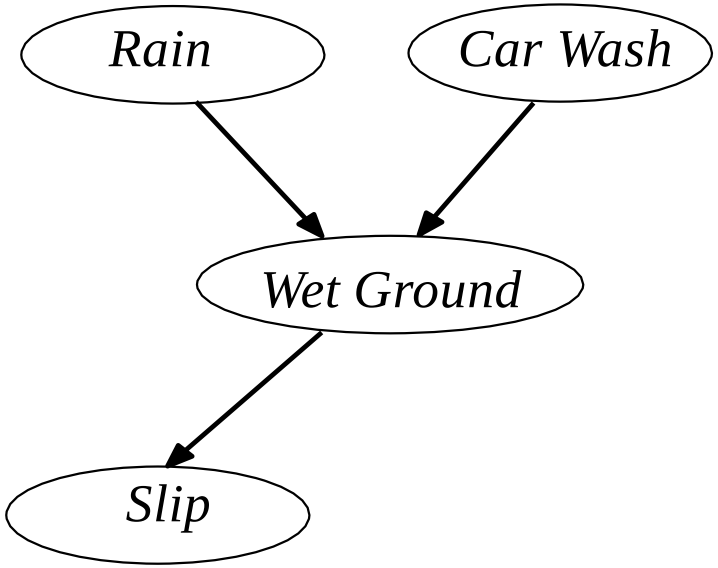
- Tips:
  - $p(X_1, \ldots, X_n) = \prod_{i=1}^{n} p(X_i | X_{pa(i)})$

- P(R,C,W,S) = P(R) P(C) P(W|R,C) P(S|W)



Worksheet: Q3- PGM # of parameters

- How many parameters in the CPTs?
  assume each variable is boolean
  (can take on one of two possible values)



Worksheet: Q3- PGM # of parameters - solution

- How many parameters in the CPTs?
  assume each variable is boolean
  (can take on one of two possible values)

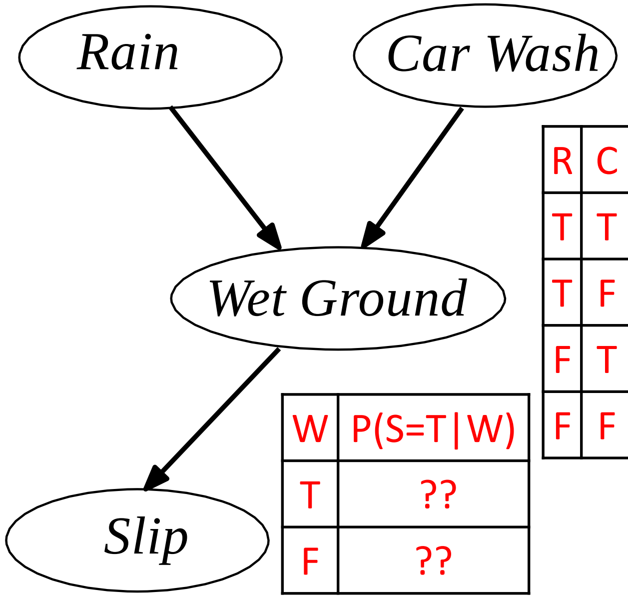
- Tips:
  - Given v nodes and if each node has 2 possible outcomes:
    - $\sum_v 2^{|pa(v)|}$

R and C have 1 parameter (prob. true)

W has 2x2 = 4 parameters

S has 2 = 2 parameters

In total 8 parameters



P(R=T)
??

P(C=T)
??

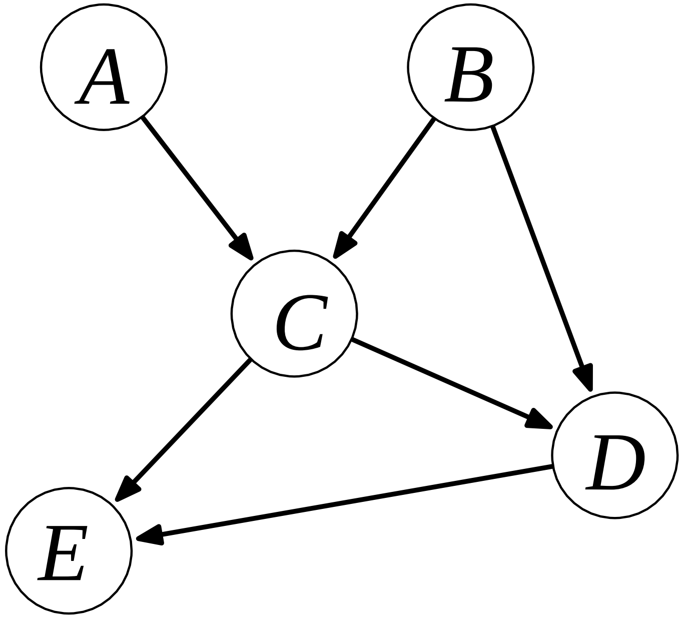
R	C	P(W=T|R,C)
T	T	??
T	F	??
F	T	??
F	F	??

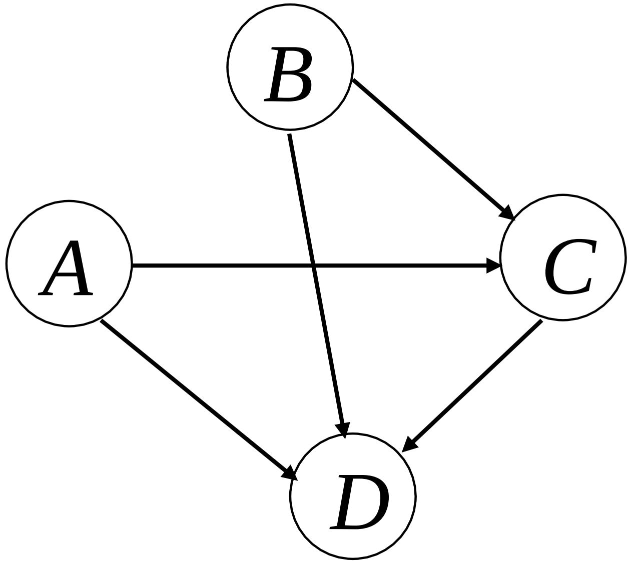
W	P(S=T|W)
T	??
F	??

P(R,C,W,S) = P(R) P(C) P(W|R,C) P(S|W)

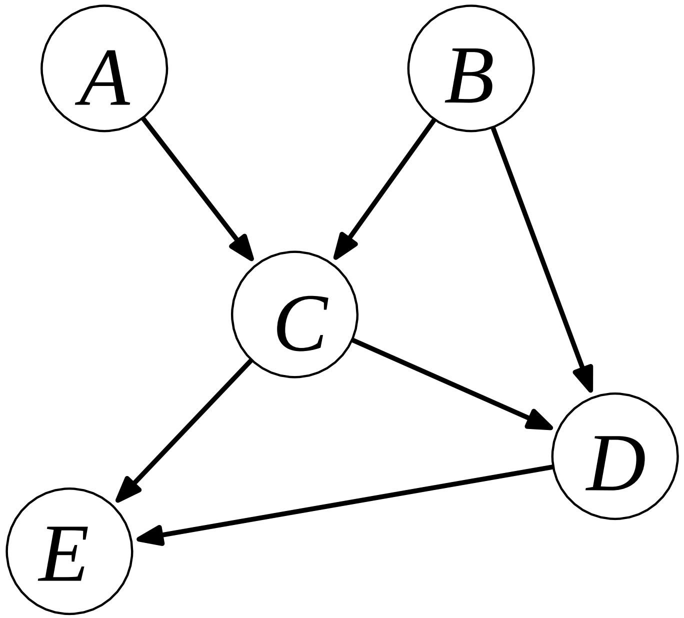
Worksheet: Q4- factorised joint distribution and # of parameters

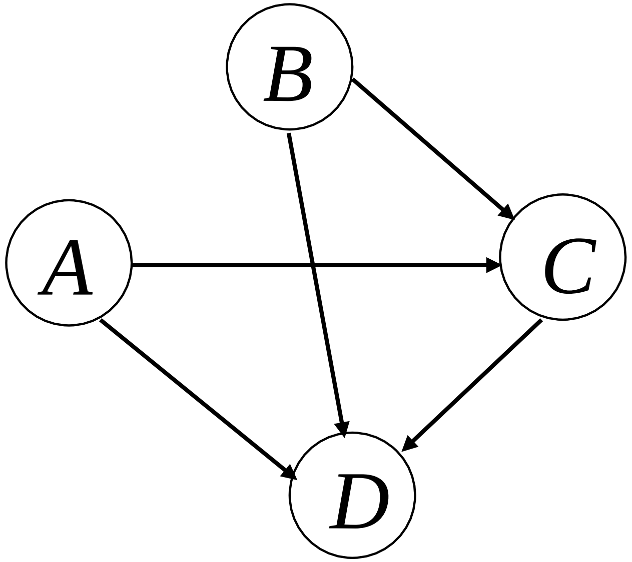
- Now repeat {2, 3} for the following two graphs.
  - Write the factorised joint distribution according to the designed graph.
  - How many parameters in the CPTs? assume each variable is boolean (can take on one of two possible values)





Worksheet: Q4- factorised joint distribution and # of parameters - solution

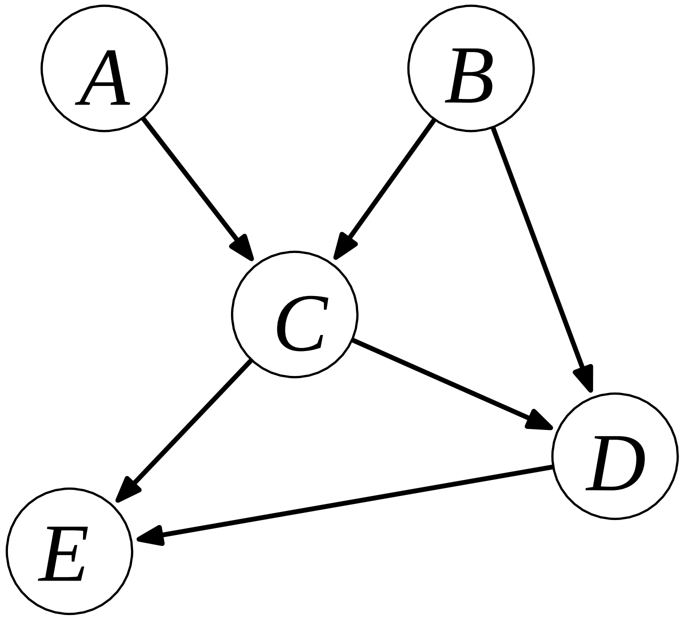


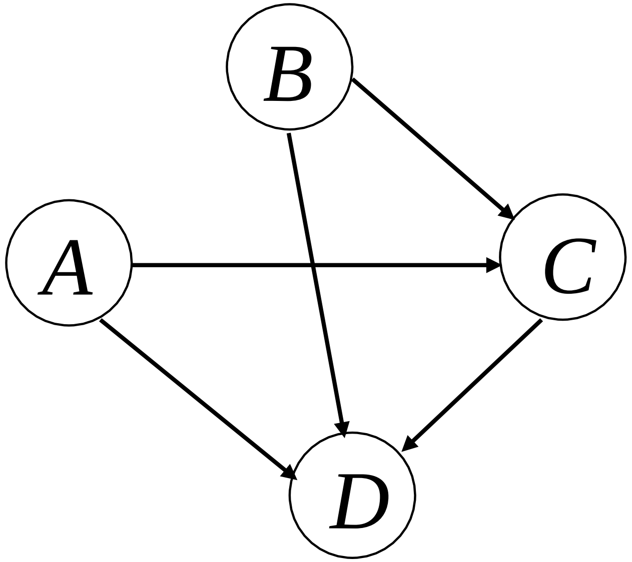


- P(A,B,C,D,E) = P(A) P(B) P(C|A,B) P(D|B,C) P(E|C,D)

- A and B have 1 parameter (prob true)
  C,D, and E have 2x2 = 4 parameters each
  In total 14 parameters

Worksheet: Q4- factorised joint distribution and # of parameters - solution





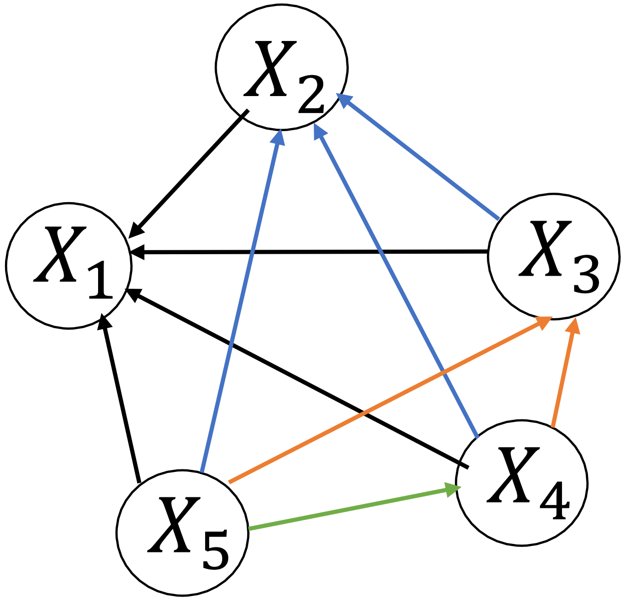
- P(A,B,C,D,E) = P(A) P(B) P(C|A,B) P(D|B,C) P(E|C,D)
- A and B have 1 parameter (prob true)
  C,D, and E have 2x2 = 4 parameters each
  In total 14 parameters

- P(A,B,C,D) = P(A) P(B) P(C|A,B) P(D|A,B,C)
- 1+1+4+8 = 14 parameters

Worksheet: Q5- full joint distribution and chain rule

- Draw a graph for the full chain rule expansion over 5 vars. How many free parameters? assume each variable is boolean (can take on one of two possible values). Write the joint probability using chain rule for this graph ?

Worksheet: Q5- full joint distribution and chain rule - solution

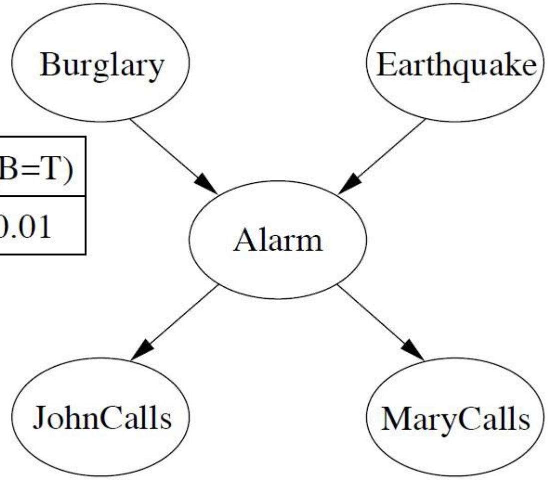


- Tips:
  - $p(X_1,\ldots,X_n) = \prod_{i=1}^{n} p(X_i|X_{i+1},\ldots,X_n)$
  - Given n: # discrete values; v: # variables (nodes)
    - Full joint PDT has # of parameters = $n^v - 1$

- $p(X_1, X_2, X_3, X_4, X_5) = p(X_1|X_2, X_3, X_4, X_5) p(X_2|X_3, X_4, X_5)\, p(X_3|X_4, X_5)\, p(X_4|X_5)\, p(X_5)$
- # of parameters: 2^5-1 = 31

Worksheet: Q6- PGMs query and conditional independence

- Given the following graph:
  - Express the *conditional* P(J=T |E=T) using mathematical symbols, and then compute the numerical values using the given CPT values.
  - Are Burglary and Earthquake independent? What about when we observe M=T?



P(B=T)
0.01

P(E=T)
0.02

B	E	P(A=T|B,E)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J=T|A)
T	0.90
F	0.05

A	P(M=T|A)
T	0.70
F	0.01

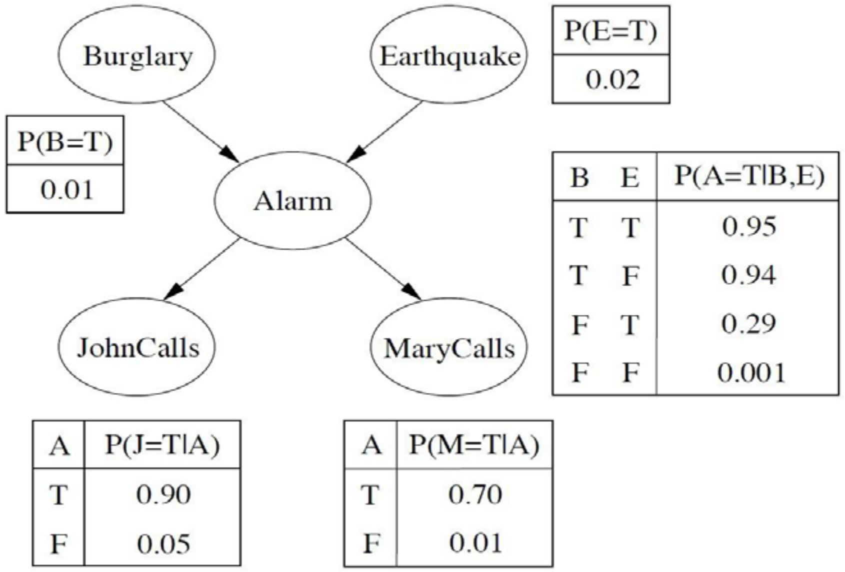
Worksheet: Q6- PGMs query and conditional independence - solution

- P(J=T | E=T) ?? Symbols form ..
- Tips:
  - Marginalisation of all possible values of *A, B and M*
  - Reorder the multiplications and push the summations inward
- $P(j\,|e) = \frac{P(\text{e},j)}{P(\text{e})} = \frac{\sum_a \sum_b \sum_m P(\text{e},j,m,b,a)}{\sum_a \sum_b \sum_m \sum_j P(\text{e},j,m,b,a)}$

- numer. $= \sum_a \sum_b \sum_m P(\text{e},j,m,b,a) = \sum_a \sum_b \sum_m$ P(b) P(e) P(a|b,e) P(j|a) P(m|a)
- = P(e) $\sum_b P(\text{b}) \sum_a$ P(a|b,e) P(j|a) $\sum_m$ P(m|a) = P(e) $\sum_b P(\text{b}) \sum_a$ P(a|b,e) P(j|a)

- denom. $= \sum_a \sum_b \sum_m \sum_j P(\text{e},j,m,b,a) = \sum_a \sum_b \sum_m \sum_j$ P(b) P(e) P(a|b,e) P(j|a) P(m|a)
- =P(e) $\sum_b P(\text{b}) \sum_a$ P(a|b,e) $\sum_j$ P(j|a) $\sum_m$ P(m|a) = P(e)

- $P(j\,|e) = \frac{\text{P(e)} \sum_b P(\text{b}) \sum_a \text{P(a|b,e) P(j|a)}}{\text{P(e)}} = \sum_b P(\text{b}) \sum_a$ P(a|b,e) P(j|a)



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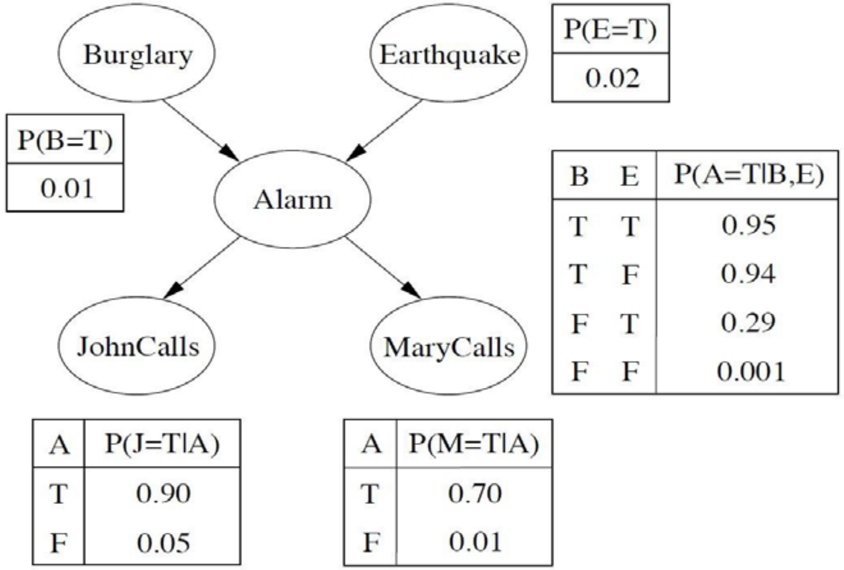
Worksheet: Q6- PGMs query and conditional independence - solution

- P(j|e) ?? Computations …
- $P(j\,|e) = \sum_b P(\text{b}) \sum_a \text{P(a|b,e) P(j|a)}$



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- Steps:
- $P(j/e) = P(\text{b}) \sum_a \text{P(a|b,e) P(j|a)} + P(\neg\text{b}) \sum_a \text{P(a|}\neg\text{b,e) P(j|a)}$
- $P(\text{b}) \sum_a \text{P(a|b,e) P(j|a)} = P(\text{b})(\text{P(a|b,e) P(j|a)} + \sum_a \text{P(}\neg\text{a|b,e) P(j|}\neg\text{a)}) = 0.01\ (0.95 \times 0.90 + 0.05 \times 0.05)$

  $= 0.008575$

- $P(\neg\text{b}) \sum_a \text{P(a|}\neg\text{b,e) P(j|a)} = P(\neg\text{b})(\text{P(a|}\neg\text{b,e) P(j|a)} + \text{P(}\neg\text{a|}\neg\text{b,e) P(j|}\neg\text{a)}) = 0.99\ (0.29 \times 0.90 + 0.71 \times 0.05)$

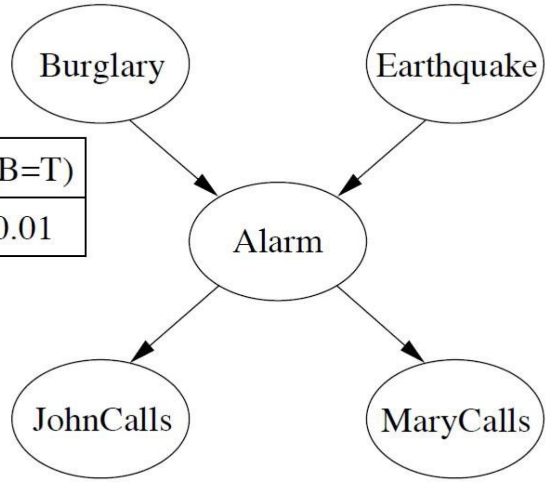
  $= 0.293535$

$P(j/e)=0.008575+ 0.293535 = 0.30211$

Worksheet: Q6- PGMs query and conditional independence - solution

- Tips:
  - Every node is dependent on its parent and nothing else that is not a descendant.

- Given the following graph:
  - Are Burglary and Earthquake independent?
  - What about when we observe M=T?



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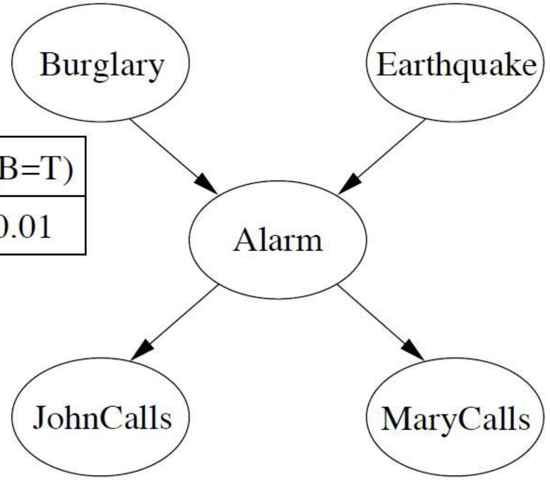
Worksheet: Q6- PGMs query and conditional independence - solution

- Tips:
  - Every node is dependent on its parent and nothing else that is not a descendant.

- Given the following graph:
  - Are Burglary and Earthquake independent?

    Yes
  - What about when we observe M=T?

    Becomes Dependent



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